

# GAME TREE REPRESENTATIONS

## Games AI Lecture 2

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### Ways of defining games

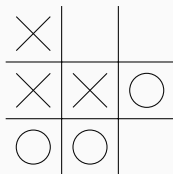
- Static vs Dynamic problems
- Deterministic vs Stochastic problems
- Fully-Observable vs Partially-Observable problems

We will be covering:

- Decision trees
- Metrics for decision trees

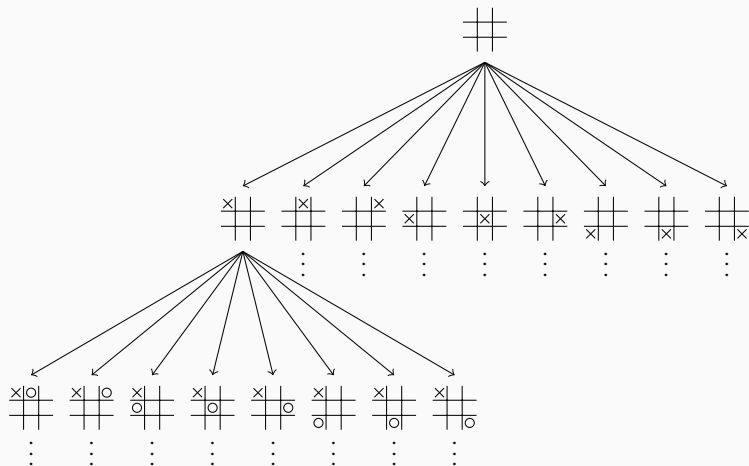
### Noughts and crosses

- Two player, deterministic, fully observable.
- Simple rules
- Most learn to play optimally when they're children

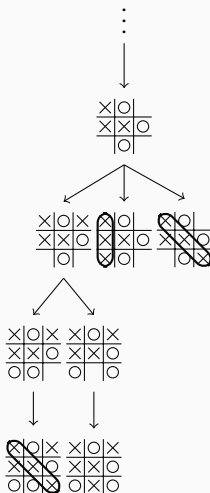




# NOUGHTS AND CROSSES - TREE REPRESENTATION



# NOUGHTS AND CROSSES - TREE REPRESENTATION



Try to construct a game tree for a simple **Subtraction Game** with people nearby in the next couple of minutes.

Aim to complete at least three layers.

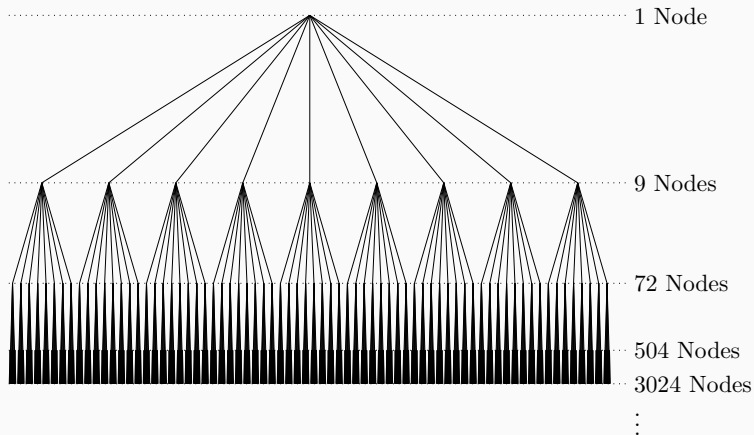
### Game Description

A counter starts with the value of 21. Two players take it turn to subtract 1, 2, or 3 from the counter. The counter cannot go negative. Once the counter reaches zero, the game ends and the player who would go next wins.



(At this point we would go through a couple of layers of the tree on the board.)

# NOUGHTS AND CROSSES - TREE COMPLEXITY



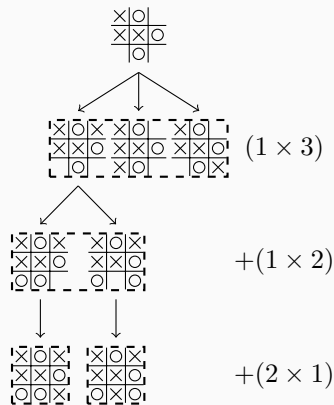
What ways can we classify the complexity of different games?

- How long the game is  
(the **depth** of the tree)
- How many actions you have to choose between  
(the **width** of a subtree)
- How many different ways the game can play out  
(the **size** of the tree)

## GAME TREE METRICS - BRANCHING FACTOR

The average number of children nodes for any non-leaf node

- Select  $n$  non-leaf nodes throughout the tree and average their number of children

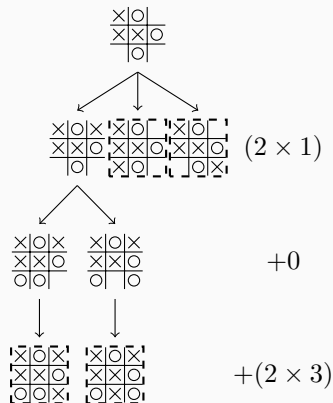


$$\begin{aligned}
 \text{BF} &= \frac{(1 \times 3) + (1 \times 2) + (2 \times 1)}{4} \\
 &= \frac{7}{4} \\
 &= 1.75
 \end{aligned}$$

## GAME TREE METRICS - TREE DEPTH

The average depth of root nodes

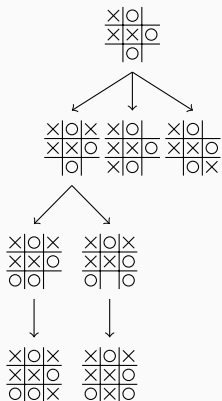
- Select  $n$  leaf nodes throughout the tree, and average their depths



$$\begin{aligned} \text{TD} &= \frac{(2 \times 1) + (0) + (2 \times 3)}{7} \\ &= \frac{8}{4} \\ &= 2 \end{aligned}$$

The number of nodes in the tree

- Count all the nodes of the full tree!
- Estimate by performing  $BF^{TD}$



$$TS = 8$$

$$TS \approx BF^{TD}$$

$$\approx 1.75^2 \approx 3.06$$

- Approximations require fair sampling
- Trees may be unbounded
- These metrics aren't everything!

- We can represent games as tree structures
- Basic tree metrics
  - Branching Factor
  - Tree Depth
  - Tree Size
- How to compute and estimate them



Before next lecture:

- **Read** “AI: A Modern Approach” Sections 3.1, 3.2  
Goes over formal definitions of games
- **Attempt** to create a game tree for the “Vacuum Domain”  
Described in the reading above!
- **Read** “AI: A Modern Approach” Sections 3.3, 3.4  
Revises simple tree searching algorithms from TPOP

Next lecture:

- How tree metrics impact searching performance
- Monte-Carlo searching techniques